

Review - Prove

$$(\sin x - \cos x)^2 - 1 = -\sin 2x$$

$$(\sin x - \cos x)(\sin x - \cos x) - 1 = -\sin 2x$$

$$\sin^2 x - \sin x \cos x - \sin x \cos x + \cos^2 x - 1 =$$

$$\underbrace{\sin^2 x + \cos^2 x} - 1 - 2 \sin x \cos x =$$

$$1 - 1 - 2 \sin x \cos x =$$

$$-2 \sin x \cos x =$$

$$-\sin 2x = -\sin 2x \quad \checkmark$$

7.5 Solving Trig Equations

LT45: I can solve trigonometric equations giving solutions as principle values, first rotation values, or all real values.

Example

$$\sin^2 x - \sin x + 1 = \cos^2 x \quad \text{for } 0 \leq x < 2\pi$$

$$\sin^2 x - \sin x + (1 - \cos^2 x) = 0$$

$$\sin^2 x - \sin x + \sin^2 x = 0$$

$$2 \sin^2 x - \sin x = 0$$

$$\sin x (2 \sin x - 1) = 0$$

$$\underline{\sin x = 0}$$

$$2 \sin x - 1 = 0$$

$$\frac{2 \sin x}{2} = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

$$0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$$

Principle values - When a variable is restricted to two adjacent quadrants.

* $\sin x$ & $\tan x \rightarrow$ Quad. I & IV ($-90^\circ \leq x \leq 90^\circ$)

* $\cos x \rightarrow$ Quad. I & III ($0^\circ \leq x \leq 180^\circ$)

Example

$$2 \sin^2 x + \sin x - 1 = 0 \quad \text{for principal values of } x \text{ in degrees}$$

$$(2 \sin x - 1)(\sin x + 1) = 0$$

$$2 \sin x - 1 = 0$$

$$\sin x + 1 = 0$$

$$\frac{2 \sin x}{2} = \frac{1}{2}$$

$$\sin x = -1$$

$$\sin x = \frac{1}{2}$$

$$30^\circ, -90^\circ$$

Example #6

$$2 \cos x - \sqrt{3} = 0 \quad (\text{Find all principal values in degrees})$$

$$\frac{2 \cos x}{2} = \frac{\sqrt{3}}{2}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$30^\circ$$